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#### ABSTRACT

Three groups of students were exposed to the same computer-administered programed instruction in numerical differentiation. Different degrees of access to the output from the typewriter terminal were given to each group. Analysis of covariance showed no significant difference on posttest scales between students who were allowed to keep the output and those who were not, nor between those students who could look back during the session at previous output and those whose view was restricted to the most recent output. (Author)

# University of North Carolina at Chapel Hill



Department of Computer Science

# A STUDY OF THE VALUE OF HARD COPY OUTPUT IN COMPUTER-ASSISTED INSTRUCTION

Ralph O. Dearbern

December 1970

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# A STUDY OF THE VALUE OF HARD COPY OUTPUT

IN COMPUTER-ASSISTED INSTRUCTION

by

Ralph O. Dearborn

A thesis submitted to the faculty of the University of North Carolina at Chapel Hill in partial fulfillment of the requirements for the degree of Master of Science in the Department of Computer and Information Science.

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Three groups of students were exposed to the same computer-administered programmed instruction in numerical differentiation with different degrees of access to the output from the typewriter terminal. Analysis of covariance showed no significant difference on posttest scores between students who were allowed to keep the output and those who were not, nor between those students who could look back during the session at previous output and those whose view was restricted to the most recent output.



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#### Chapter I

#### THE PROBLEM

# 1.1 Overview of Computer-Assisted Instruction

# 1.1.1 Modes of Use

The application of the relatively new technology of computers to the ancient problem of learning has taken many forms. These can be classified, on the bases of purpose and means of achieving that purpose, in five distinct categories,

- (1) As a student learning tool, the computer is used for problem solving. The student may write programs or use prewritten programs which are at his disposal. This mode is characterized by the control which the student exercises over the use of the tool.
- (2) As a teaching aid, the computer can be used in a manner similar to chalkboard, slides, films, etc. The ability to access a computer from a classroom terminal allows an instructor to give examples which may otherwise be omitted due to time constraints or tedious computations. This permits demonstration of, for instance, convergence of an iterative process, repeated calls of a recursive function, or the effect of various reentry speeds on the landing site of a space vehicle.



- (3) As a simulation device, the computer can provide students an opportunity to apply principles and concepts learned earlier. The most familiar example is the Sumerian game, in which the student assumes the role of the King of Sumer. The actions he takes to solve various social, political, and economic crises determine the future of the country.
- (4) As a tool for instruction management, the computer is used for storage and retrieval of teaching materials and student records. Individual student lessons can be prescribed by analysis of past performance.
- (5) The most commonly held image of computer use in the instructional process is that of a student seated at a terminal, being taught by a program over which he has little or no control. This author-controlled mode is used for drill, testing, and the presentation of new material.

# 1.1.2 <u>Costs</u>

The above modes of computer use share some common problems, not the least of which is cost. There seem to be four
main costs involved: central processing unit (CPU), terminals, communications, and software. Regardless of whether
the computer is owned or leased, or CPU time is bought from
someone else, the cost is very real. Terminal costs vary
widely depending upon the type of terminal, and the mode of
use determines the number required. Certainly, to involve a



significant number of students in simulation, authorcontrolled, or problem solving mode requires multiple ter-Terminal costs are discussed more fully in Section minals. 1.3.8. Economic considerations may require one CPU to be accessed by widely scattered terminals, thereby introducing the cost of communication among the several locations. Depending upon the number of terminals, their geographic distribution, and the means of communication, the perstudent-hour operating cost of such a system is likely to be less than that incurred using multiple independent computers. The currently most common means of communication is via commercial telephone lines. Finally, there is the cost of buying, leasing or writing the software necessary to utilize the hardware. This includes the system support software as well as the instructional programs. As an indication of the magnitude of the cost of the latter, it is variously estimated that 100-200 man-hours are necessary to prepare one hour of author-controlled instruction [4, pp. 85-89].

# 1.1.3 Narrowing the Scope

The focus of this investigation is the author-controlled mode. All subsequent references to computerassisted instruction (CAI) should be interpreted as author-controlled presentation of new material to a single student at a terminal. Feldhusen and Szabo [15] give a good survey



of the many studies which have been made of learning characteristics, branching strategies, methods of reinforcement, stimulus presentation, and other program dependent features of CAT. This study is concerned with the effect of characteristics of the terminal itself upon learning.

## 1.2 Common Peatures of Terminals

# 1.2.1 CPU Requirements

To be of use in CAI, a terminal must be able to access a central processor. The demands it places upon the CPU vary with the type of terminal and its major use. For example, an unbuffered volatile cathode ray tube must be continually refreshed by the CPU. The CPU may be dedicated entirely to the CAI system or, as is often the case, may operate interactively with the student user while doing batch processing in the background.

#### 1,2,2 Keyboards

Human-to-computer communication is achieved in most instances via a keyboard, similar to that of an ordinary typewriter. The keys, used singly or in combination, represent alphabetic, numeric, or special characters. Among the special characters are such symbols as + @ and #, and control characters which govern the operation of the terminal and the CPU. As discussed in Section 1.3.6, many video display terminals have other input mechanisms.



# 1.2.3 Output Display Media

The computer-to-human communication can take many forms, some generated by the computer, others simply under computer control.

The vast majority of these forms are visual in nature, although some work has been done using random access tape recorders, under computer control, to provide audio stimulation.

The display media required on a terminal depend to a certain extent upon the intended use. For instance, the ability to provide audio output would be desirable in teaching a foreign language and nearly imperative in dealing with very young children, but of questionable use in teaching numerical analysis to college students. Similarly, devices capable of producing high resolution graphic displays are not needed if the only required output is text.

It appears, then, that if one has need of a specialized terminal, knows the capabilities required, and is willing to spend the necessary money, one can obtain the terminal.

Most CAI practitioners are not in that position. They either want a general-purpose terminal, are not sure what their needs are, operate under a tight budget, or, quite likely, fit all three descriptions. The problem is then reduced to a choice between two basic types of terminal, the typewriter style and the cathode ray tube.



# 1.3 Typewriter Terminals versus Cathode hay Tube Terminals

# 1.3.1 Definitions

Let us consider a "standard" CRT terminal to include a display screen, alphanumeric keyboard, and light pen. Feference to CRT's will imply all terminals of this type, ione such terminal is the CC-30 Display Station from Computer Communications, Inc.) Reference to TWR's will imply the broad class of typewriter-like terminals, including the Teletype ASR models and the IBM 2741. Comparing the two types, we note the following differences.

#### 1.3.2 Speed

The physical movement of the typing element limits the print speed of TWR\*s to 10-15 characters per second, whereas the display rate of a CRT is limited in practice only by the transmission rate. When one uses low speed communication lines the two types of display occur at approximately the same rate. Wodtke and Gilman [16] suggest that a display rate significantly slower than a student's reading rate can produce boredom and subsequent avoidance of CAI as a means of learning. Typical reading rates for college students (300-400 words per minute) are two and one-half to three times the print rate of TWR\*s.

#### 1.3.3 Noise

The noise level of a CRT is negligible, whereas a TWR produces harsh, staccato sounds caused by carrier returns, case shifts, type element rotations, and printing impact.



The noise problem is particularly acute when the terminal is used in a small room with poor acoustics or when many terminals are located in one room.

# 1.3.4 Reliability

The typing mechanism of a TWR is electromechanical, whereas the display production in a CRT is electronic. The susceptibility of the moving parts to maladjustment and damage makes the TWR less reliable than a CRT in terms of both frequency-of-repair and correct display of characters [3]. In both types of terminals, the keyboard used for input is electromechanical, but this is a much less frequent source of trouble than the output mechanism.

# 1.3.5 Graphics Capability

In order to plot a graph on a TWR terminal, one chooses one of the available characters and places that character in selected print positions. The spacing of the type element over many blank positions causes a potentially annoying delay. Moreover, the characters can be no closer than one line apart vertically and/or one print position horizontally. This may be entirely acceptable in representing discrete data but offers only a crude approximation to continuity.

Most CRT's, on the other hand, produce an alphanumeric character by illuminating selected points within a rectangual lar dot matrix. If the points can be illuminated individually, as some terminals allow, rather than as part of a character, they can be used for producing graphs, maps, and



diagrams of greater resolution than is possible with a TWR.

Moreover, the rapid display presents the picture to a user as a conceptual entity rather than a series of isolated characters. This graphic capability allows the CAI author a wider choice of subject matter and presentation mode.

#### 1.3.6 Response Mode

To use a TWR terminal, a student must, at the very least, be able to find a given letter on the keyboard, strike the key, and strike the return key. Thus the non-typist, especially the young child, is penalized in terms of time spent entering responses. The insecurity and distraction associated with the "hunt and peck" method may interfere with the student's concentration on the subject matter.

On the typical CRT the electron-detecting light pen can be used to select, by merely pointing, a response from a multiple choice list displayed on the screen. Swets [15] has used the CRT to provide a two dimensional interpretation to responses which were linear on a TWR. (See Section 1.5.1.) Combined with the graphics capabilities discussed in the previous section, the light pen greatly increases the variety of teaching methods available to CRT users.

# 1.3.7 Error Correction

On a TWR, a typographical error is overstruck, corrected on the line below, or "erased" by typing a special character once for each character entered after the one in error.

There may be a negative effect upon the user when the in-



correct character remains in place. On a CRT, when the offending character is overstruck, the original character is replaced by the most recently entered character.

#### 1.3.8 Cost

As would be expected, the cost of a terminal depends upon a multitude of factors. A CRT of the type under consideration can be purchased for \$6000-\$9000. The variance in price is due to features such as storage buffers, character sets, display size and resolution, display modes (e.g. graph or character), etc.

Within the TWR classification there is also a choice of features, such as maximum line width, interchangeable type elements, special character sets, print speed, noise, etc. A Teletype Model 33 ASR can be purchased for less than \$1000, whereas an IBM 2741 may cost \$4000.

The importance of the cost factor is reflected in the statement by Dr. Patrick Suppes in 1966 that ". . . we'll probably be using mostly teletype and typewriter stations for the next five years for the simple reason of economics" [13].

#### 1.3.9 Hard Copy

TWR's produce a record on paper of a student's entire session at a terminal, while CRT's in general produce no such record. The following section deals with this hard copy and its use.



# 1.4 The Role of Hard Copy

# 1.4.1 During the session

Because of the transitory nature of the display on a CRT, the student does not have a record of his preceding work. Assuming a sequential presentation of subject matter, it may be instructive for the student to review, at any time he wishes, some particular point or the over-all development of the lesson. With hard copy output this can be done simply by looking back at the pertinent sections of the printed record. With a CRT, unless special measures are employed, the record is lost when it is removed from the display screen. Several possible means of overcoming this difficulty are discussed in Section 1.4.3.

# 1.4.2 After the Session

Upon leaving a session at a TWR, the student can take along a complete sequential record of the lesson, including material presented, requests for input, his own responses, program reaction to his responses, data summaries, etc.

Depending upon the subject matter, the student's age, the instruction mode, and various external factors, this record may be used in any of several ways.

(a) Comparison of the output with other students may give an understanding of why certain responses were incorrect. Comparison of results on quizzes administered during the session may provide the peer approval or competitive urge needed to encourage a particular student.



- (b) Parental inspection of the output could increase parents' understanding of the nature of computer-assisted instruction. Such inspection may also prove to be valuable incentive to students to perform well.
- (c) One of the greatest potential benefits of CAI is individualization of instruction. Utilizing data on an individual student's past performance, the teaching program could generate a homework assignment tailored to the particular student's needs. This could be printed on the hard copy output and taken home.
- (d) Perhaps the most likely and most valuable use of hard copy is as a study guide for the student himself. Time constraints at the terminal often prevent the student from studying the development of the lesson as deeply as he would like. There may also be specific points which the student did not understand and which he would study further if he had a record of the lesson at that point. If no printed record of the lesson is available, further study of the lesson itself is not likely.
- (e) As one means of further study, the student could take the original to an "authority", be it the teacher involved or some one else, for a conference concerning the material presented in the lesson. The hard copy would be especially valuable in the case that the "authority" was not familiar with the lesson.
  - (f) Aside from student use, the printed record could



be used as a diagnostic and by the teacher who could review the student's entire terminal session on a reduced time scale at the teacher's convenience. The CAI program author could use output from the student sessions to look for program errors, extremely difficult parts, ambiguities, etc.

#### 1.4.3 Alternatives

Some of the needs for hard copy mentioned above can be met by a <u>partial</u> record of the session. This can be provided in several ways other than having a TWR print the entire session.

When a CRT is used, the program author is aware that hard copy will not be created and can alter his program accordingly. Important points and summaries can be stored and made available for display upon student request. It is conceivable that enough information could be kept to recreate the session later, but this would require vast amounts of storage and its value is not clear.

Another possibility is use of a terminal which supplies both the advantages of a CRT and a hard copy mechanism.

Many CRT's in the sophistication and price range under consideration can be equipped optionally with printers. The cost naturally depends upon the speed, quality, and versatilaty of the printer. The CC-30, as one example, can incorporate a Teletype 33 Read Only Printer that sells for approximately \$600. Other terminals are available which are



basically CRT's, but which can produce, by thermal or photographic means, a hard copy reproduction of the image currently on the screen.

The question then arises as to what to print. This choice could be made by the program author, who knows the relationships which exist between various parts of the program and who may speculate on what portions the student may want printed. Or the choice could be left to the student, who knows what he wants but may not always know what he needs.

one further possible solution is to store on disk whatever amount (possibly all) of the session one desires to list, then have all session records printed on a common printer. The listings could then be distributed physically to the students or teacher. Such an approach is taken with the IBM 2848 Display Control system, in which the controller can handle multiple 2260 Display Stations and one 1053 Model 4 Printer.

of the possible uses of hard copy specified in Section 1.4.1 and Section 1.4.2, one sees that these hybrid devices can satisfy some better than others. The greatest difficulty lies in anticipating what use will be made of the hard copy and recording the pertinent portion of the session.

Nothing short of printing the entire session will serve all the needs completely.



#### 1.5 Definition of the Problem

#### 1.5.1 Related Research

As mentioned earlier, many comparative studies have been made, with widely varying results, comparing computerassisted instruction, programmed text, and classroom lecture. Other investigators have compared instructional strategies, such as remedial vs. forward branching, differing degrees of feedback (prompting), and varying amounts of student control over the lesson sequence. These studies have been directed toward devising an optimal instructional program and are not directly related to the study herein described.

Relatively few investigators have addressed themselves to the question of machine-to-student communication. Glaser, Lipson and Ramage [6] have attempted to characterize the interface between student and subject matter, an interface which naturally includes the physical communication devices. Their major concern is the specification of techniques for optimal utilization of the visual and auditory senses of students. Consequently, their recommendations for developing devices for maximum flexibility fall primarily in the "long range" category.

Woodtke and Gilman [16] found that college students working in a programmed text and students exposed to the same sequence administered by computer through a typewriter terminal did not achieve significantly different scores on a posttest. The computer group did take considerably longer



to complete the lesson. The investigators concluded that the typewriter print speed was slower than the students' reading rate, suggesting that the typewriter may be more appropriate for relatively nonverbal subject matter.

Johnson and Borman [8], taking their cue from Wodtke and Gilman, exposed 90 college students to a CAI lesson in basic physics under one of four presentation modes: audio, display, type, slide. In type mode, the typewriter terminal was the sole interface device. In each of the others, the typewriter was used for student entries and some information display. Lengthy displays were presented with photographic slides, a pre-printed workbook, or audio tape messages. No significant difference was found in either posttest scores or completion times of the four treatment groups.

Swets and others [14] were concerned with teaching subjects to identify sounds which could vary in frequency, amplitude, percent of on-time, interruption rate, and length. Their experience led them to believe that the typewriter might be an inconvenient means of communication. In later studies [15] dual experiments were conducted, a typewriter being used in one and a CRT in the other. Each sound was "named" by a five digit number, each digit representing on a 1-5 scale the value of one of the five dimensions listed above. In the TWR experiment, the student had to name the sounds he heard by typing in this five digit number. Comparison of incorrect and correct names required comparison of two numbers. In the CRT version of the experiment the



naming was done by pointing a light pen at intersections of a 5 % 5 jrid matrix whose horizontal units were the five dimensions and whose vertical units were the values of each dimension. Thus a spatial interpretation was established between, for instance, high pitch and higher points of the matrix. Comparison of incorrect responses with correct names was done by matching x's and o's on the matrix. This apparently more convenient means of response and feedback did not produce significantly better results on the post-treatment test.

# 1.5.2 Focus of the Study

One of the most urgent tasks in CAI research is to determine the circumstances under which each of the many modes of machine-to-student communication is effective. Of the studies cited above, only the last involved the two most widely employed types of CAI devices, the typewriter and the cathode ray tube terminals. Even in Swet.'s work the terminal was of seemingly minor importance, since the information presented was codified and always in the same format.

The original idea for this study was to compare a type-writer terminal (the Datel Thirty-21, which is very similar to an IBM 2741) and a CRT terminal (the CC-30 from Computer Communications, Inc.) as the sole interface with a student in administering programmed instruction. When one looks at the differences enumerated in Section 1.3, however, it is not difficult to imagine that some have little bearing on reten-



tion of subject matter. Cost, for example, is a concern of those responsible for providing the interface but not a factor directly involved in the instructional process. Except for an annoyance factor, reliability is in much the same category. The other differences, with the exception of the lack of hard copy, are favorable toward the CRT. Thus if cost is not a major obstacle, the only potential reason for not using CRT's is the absence of hard copy output.

The author therefore decided to analyze the value of hard copy, both during and after the terminal session.

Because of the interests of the sponsoring organization and the environment in which the investigation was to be conducted, the study was focused upon the teaching of numerical analysis to upper level college undergraduates and beginning graduate students. The instructional mode chosen, for reasons stated in Section 3.4.1, was computer-administered programmed instruction with limited prompting available upon student request.



#### Chapter II

#### THE STUDY

#### 2.1 Procedure

Twenty-three students at the upper undergraduate-lower graduate level were divided randomly into three test groups. All subjects were given a pretest designed to measure their knowledge of the specific subject to be presented and their grasp of the mathematical ideas upon which the presentation was based. Each subject was exposed to the same computeradministered programmed instruction sequence teaching numerical differentiation at a typewriter terminal. Members of the control group, hereinafter referred to as Group 1. used the terminal in the normal manner and were instructed to take the printed record of the session with them upon leaving. Members of Group 2 operated the terminal in the same manner but were not allowed to remove the printed output from the terminal room. For members of Group 3, the terminal was fitted with a box-like device which allowed the user visual access to only the most recent part of the output. Such a view is similar to what one would see on a CRT screen. (Details of the physical arrangements are given in Section 2.5.) Two days after the last student underwent treatment, a posttest was given. Analysis of



covariance was used to determine whether the group scores were significantly different. Similar tests, of secondary interest, were performed to determine whether the groups varied significantly in other respects.

# 2.2 Subjects

# 2.2.1 Characteristics

The subjects were all enrolled in the course COMP 150. Introduction to Numerical Methods, offered by the Department of Computer and Information Science of the University of North Carolina at Chapel Hill. The course is taken by some beginning graduate students in the same department as partial fulfillment of the requirements for a Master of Science degree. Other students are primarily graduates or advanced undergraduates majoring in the physical sciences who are interested in applying numerical techniques within their discipline. Table 2.1 shows the academic fields of interest of the subjects by group; Table 2.2 shows the academic year in which the subjects were classified; Table 2.3 gives a sketchy profile of the subjects academic backgrounds in mathematics. The data shown in the tables were gathered by . a questionnaire given to the students at the time of the posttest.

Some qualitative observations can be made on the composition of the groups. Group 2 may be characterized as being further advanced in their studies and, by the author's



observations, older than the other subjects. The same group appears to differ from the others in area of major interest. Especially notable is the absence of Computer and Information Science majors and the presence of three biological scientists. One might reasonably predict that the members of Group 2, having been in their field of specialization longer and therefore farther removed from their formal mathematics training, would have more difficulty with the lesson. From data gathered by the teaching program itself, one finds that only 42% of Group 2 had used a computer terminal before, compared with 50% and 63% of Groups 1 and 3, respectively.

It should be noted here that of the 23 subjects, two did not return the questionnaire, two others did not take the pretest, and one did not take the posttest. All statements in this report referring to a group should be interpreted as applying to that subset of the group for which data are available.

	Group 1	Group 2	Group 3
Computer Science	2	0	3
Bioengineering	0	1	0
Biology	0	2	1
Chemistry	2	0	0
Engineering	1	1	0
Geophysics	0	1	0
Mathematics	1	1	ı
Physics	2	1	1

Table 2.1 Subjects' academic majors by treatment groups



	Group 1	Group 2	Group 3
Junior	1	0	0
Senior	1	1	1
First year graduate	3	0	3
Second year graduate	1	2	2
Third year graduate	1	3	0
Other	1	<u>1</u>	0

Table 2.2 Subjects' academic classifications by treatment groups

	Group 1	Group 2	Group 3
Semester hours credit		,	
(a) mean	18.5	15,6	13.6
(b) range	12-30	3-40	6-24
Cumulative average			
A	2	1	3
В	4	5	2
С	2	1	1

Table 2.3 Subjects' mathematics backgrounds by treatment groups



## 2.2.2 Motivation

The experiment was presented to the subjects as a CAI project to determine the effectiveness of some new teaching materials. The subject matter was related to, but not dependent upon, the students' current topic in the course. Emphasis was placed upon the consistency of the teaching program with the students' text in both content and notation. Students were asked to participate in a terminal session at their convenience in lieu of holding class on a particular day. The pretest, given without notice, and the posttest, which was announced at the time of the pretest, were given during regular class time. The students were informed that their posttest score would be counted as one of approximately 16 assignments but that no grades of any type would be given on the basis of their responses during the terminal session. It was felt that the above approach would provide sufficient motivation without inhibiting the students responses to the lesson.

Students were not informed of the three output treatment groups until after the posttest was completed. Instructions concerning the disposition of the printed output were
given by the teaching program itself, so that no indication
of differing treatments was present.



## 2.3 Subject Matter

#### 2.3.1 Selection

The choice of subject matter for the experiment was influenced in large measure by the environment in which the study was to be conducted. The research was under the auspices of the Department of Computer and Information Science, whose students were readily available as subjects. It was felt that avoidance of the computer as subject matter would prevent potential confusion with the computer as a teaching device. The availability of a series of CAI programs in numerical analysis further influenced the decision. The series was designed as a semester course whose content and approach parallel that of the textbook used in a course offered by the Computer and Information Science Department. The particular lesson was chosen because

- (1) the subject seemed appropriate for the students' expected progress by the date set for the experiment;
- (2) the professor did not intend to lecture on the particular topic in class;
- (3) the mathematical development of the lesson was relatively independent of other specific knowledge.

# 2.3.2 Content

The objectives of the lesson were

- (1) to introduce the concept of the order of an approximation and the notation  $O(h^{k})$ ;
- (2) to derive specific formulas for 0(h) and  $0(h^2)$



24

approximations to  $f'(x_i)$ ;

- (3) to derive a specific formula for an  $O(h^2)$  approximation to  $f''(x_i)$ ,
- (4) to demonstrate the effect of roundoff error upon the foregoing approximations.

The lesson was divided into the following seven sections, each dealing with one subtopic:

- A. Order of an Approximation
- B. Functions Tabulated on an Equally Spaced Set of Points
- Cl. Order h Approximation to f'(x)
- C2. Order h Approximation to f'(x)
- C3. Order  $h^2$  Approximation to  $f'(x_0)$  and  $f'(x_N)$
- C4. Order  $h^2$  Approximation to  $f''(x_i)$ , 0 < i < N
- D. Computational Accuracy of Numerical Differentiation An outline of the material presented is included as Appendix A.

Throughout the lesson, development of approximation formulas was based upon Taylor expansions without remainders, an approach slightly different from that of the students' textbook. All results and notational conventions were consistent with the text. Continual emphasis was placed upon the error in the approximations and the assumptions of continuity and differentiability of the functions whose derivatives were to be approximated. Section D gave students a chance to observe the  $O(h^2)$  approximations to f'(x) and f''(x) as the interval size approached zero, and the effect of



roundoff error.

#### 2.4 Teaching Program

#### 2.4.1 Development

The program was developed and tested by Dr. Arthur E. Oldehoeft of Purdue University as part of a complete course in computational mathematics, pursuant to a grant from the Office of Education, U. S. Department of Health, Education and Welfare [10]. A comparable course is taught at Purdue by conventional methods by Dr. S. D. Conte, whose textbook, Elementary Numerical Analysis [3], is the primary reference both of the programmed course and of COMP 150. The programmed course, consisting of twenty-five lessons, was written in the language PICLS (Purdue Instructional and Computational Learning System), which operates interactively under the MACE Operating System on a CDC 6500 computer at Purdue. Student terminals used for initial program testing at Purdue were KSR-33 Teletypes.

A large portion of the development effort was devoted to writing and testing a function matching routine which allowed students to enter constructed mathematical expressions in response to questions. The matching routine checked the syntax of the student's expressions and tested them for equivalence with expressions specified by the



Now at Iowa State University.

author. This allowed the student maximum flexibility in entering responses. The routine that performed this matching function was installed as part of the PICLS system, not as part of this CAI program.

The use of Teletype terminals restricted the notation to a strictly linear form, making lengthy expressions cumbersome. Such expressions were avoided where possible by use of multiple choice questions. Similarly, the use of graphs and diagrams was avoided because of the slow print speed and low resolution of the terminals.

# 2.4.2 Local Adaptation

Since the facilities available to this author did not include PICLS, extensive modification of the teaching program was necessary. For a variety of reasons, including availability, suitability to the task, and the investigator's personal preference, the program was translated into APL\360 [7]. Many of the vital portions of the original program were imbedded in the operating system rather than the program; comparable operation was provided through author-defined APL functions. No attempt was made to recreate the function matching routine because the particular lesson being used required only three constructedexpression responses. These were eliminated by changing the corresponding questions to a multiple-choice format. When a question called for a numeric response, however, students could enter expressions without reducing them.



example, applying the approximation formula

$$D(h) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h} \quad \text{where } f(x_{i+1}) = .853,$$

 $f(x_{i-1}) = .603$ , and h = .1, the student could enter  $(.853-.603) \div (2 \times .1)$ , which would be evaluated and compared with the right answer. It was intended that this would relieve the student of performing the arithmetic by hand, but because of the students' lack of familiarity with the method of expression evaluation in APL, few students used the facility.

Another problem which arose in the adaptation was that of capturing data on student responses, working time, etc. In the PICLS version, such records were saved automatically by the system. In APL, system commands such as copying data into and saving data from an active workspace cannot be included in an author—defined function. It was therefore necessary to ask the student to perform a sequence of four or five system commands after eight of the nine sections of the lesson. Once again the students lack of familiarity with APL caused some data to be lost and some to be recorded incorrectly.

Since one of the treatment groups (See Section 2.1.1.) was not allowed to look back over the output, an attempt was made to eliminate the necessity of doing so. All tables and formulas were repeated where necessary to make them visible within the most recent 40 lines of output.



Although extensive internal revision was done, the program remained basically unchanged in outward appearance.

The advantages of a well planned lesson, written in collaboration with the textbook author and tested in an actual course, were retained.

#### 2.4.3 Teaching Strategy

#### 2.4.3.1 General

There were nine independent sections in the lessons, two of which (BEGIN and SUMMARY) were not directly concerned with presentation of subject matter. Six of the remaining sections, identified as PARTA, PARTB, PARTC1, PARTC2, PARTC3, PARTC4, used a linear teaching strategy in which the student was typically presented some text, then asked a question whose answer was a direct outgrowth of the text. The material was not presented in the small-step fashion often associated with programmed texts (e.g. "Fire engines are red. What color are fire engines?"). Rather, making an intelligent response required completing a step in the derivation of a formula, applying a formula to a given set of data, or some comparable process.

A correct student response brought an acknowledgement of correctness and the next text or question. If a student entered an incorrect answer, he received a message, usually "NO. TRY AGAIN OR TYPE 'HELP'.", and was asked the same question again. A right answer on the second try was



treated in the same manner as on the first try. A second wrong response, however, caused the correct answer to be printed, along with the reasoning leading to that answer.

In lieu of an answer to the question posed, a student could type either "HELP" or "STOP". The former was a request for a clue. The program responded with some message clarifying the question or suggesting an approach to finding the answer. An entry of "HELP" was not counted as a response in determining when two wrong responses had been made. By typing "STOP" in response to a question, the student was able to leave the section in which he was working. He could then sign off or begin any other part.

#### 2.4.3.2 Functions ASKMC and ASKNUMERIC

The logical sequence required to implement the above strategy was contained in two functions, ASKMC and ASKNUMERIC. Two routines were necessary because of the fundamental differences in the ways APL\360 handles character and numeric data; but the legical flow, as shown in Figure 2.4, was the same for both functions. The functions themselves are shown in Appendix B.

One of the two functions ASKMC or ASKNUMERIC was called by the main function (i.e., the lesson section) after some global variables had been set. (Global variables are those whose value is "known" both within and without the function being called.) The variables and their uses are as follows:



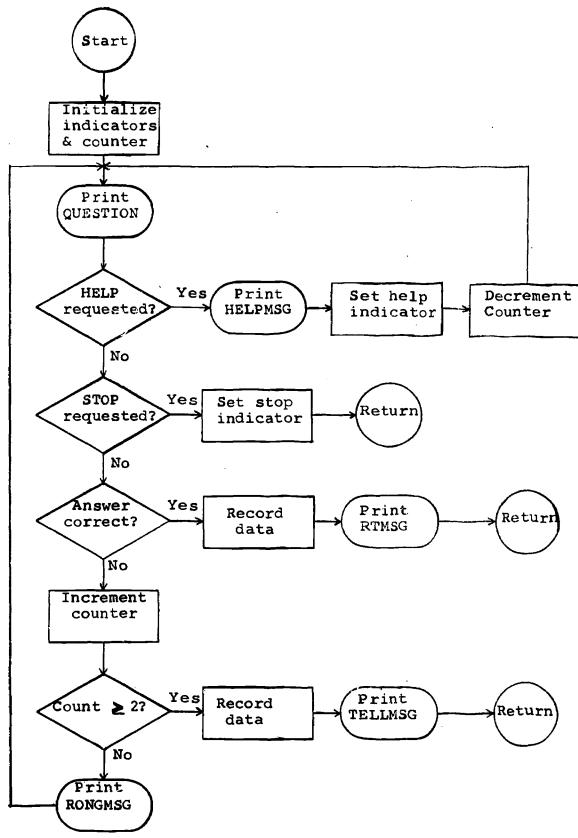


Figure 2.4 Common flow of functions ASKMC and ASKNUMERIC



QUESTION: A vector of characters used to pose the question.

RTMSG: Vector of characters; message to be printed when student responded correctly; usually "RIGHT".

RONGMSG: Vector of characters; message to be printed when student responded incorrectly; usually "No. TRY AGAIN OR TYPE 'HELP'."

HELPMSG: Vector of characters; message to be printed .

when student requested help.

TELLMSG: Vector of characters; message to be printed after two incorrect responses; consisted of the correct answer and some explanation.

RTANS: Either a number or a vector of the unclers: the correct response.

Note that RTANS may be either numeric or character, depending upon the question to be posed. The choice of function to be called was necessarily consistent with the type of data assigned to RTANS.

Several differences existed between ASKNC and ASKNUMERIC. The former was used primarily with multiple choice questions. In the case of only two possible choices, it seemed unwise to ask the question again after receiving one incorrect response. The student would, in all probability, respond correctly the second time and proceed to the next question with no explanation of his error. A more satisfactory solution would be to tell him he was incorrect, explain why, and



proceed with the lesson. To accomplish this, a parameter was passed to ASKMC giving the number of possible choices presented to the student in QUESTION. This number was used in initializing the counter which was used to determine when to tell the student the answer. The net result was that with two choices, one response was allowed, whereas with three choices, two responses were allowed. There were never more than three choices. For questions requiring numeric responses, the student was given the correct answer after two unsuccessful attempts.

A second difference was the method of checking for a valid, or reasonable, response. No check is shown in Figure 2.1 since the two functions handled the problem in very different ways. ASKNUMERIC was used only when the student was expected to respond with a number or an arithmetic expression. APL\ 360 interpreted any alphabetic response as a variable to which no value had been assigned and printed the message "VALUE ERROR", followed by the student's response. It was the author's opinion that this was sufficient explanation, so no other error-checking capability was included. In ASKMC, no system capabilities provided the error-detecting mechanism desired, so the function was designed to do some minor screening of answers. If the student's response was neither "HELP" nor "STOP" nor one letter long, the student received the message "YOU MUST CHOOSE ONE OF THE GIVEN ANSWERS OR TYPE 'HELP'." A one-letter response which was



valid, such as "X" when the choices were "A", "B", and "C", was treated in the same manner as a valid, but incorrect answer. In neither function was an invalid response counted as an incorrect response for branching purposes.

Another function, PAUSE, was called by both ASKMC and ASKNUMERIC, as well as by the lesson session main functions, to provide a variable length delay in the lesson to allow the student some "contemplation time". The message "TYPE 'P' TO PROCEED" was printed and execution was suspended until the student did so. PAUSE was used most regularly after TELLMSG was presented, allowing the student to analyze his incorrect responses and the correct one. PAUSE was also used after any long section of text, particularly those with a substantial amount of mathematical notation. A sample session, taken from PARTC4, is shown in Figure 2.5. The APL\360 program for the same portion is shown in Figure 2.6.

# 2.4.3.3 PARTD Strategy

The strategy employed in the last teaching section was markedly different from that of the other sections. It was an attempt to incorporate features of what Oldehoeft calls the Problem Mode and Investigation Mode. After several questions in the conventional mode, the student realized that for  $f(x) = e^{X}$ ,  $f'(x) = f''(x) = e^{X}$ , and that f'(0) = f''(0) = 1. Two approximation formulas which were derived earlier were presented to the student. One forumula,

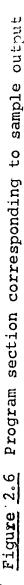


```
SO OUR APPROXIMATION FORMULA FOR THE SECOND DERIVATIVE IS
**
** D2(H) = (F[I+1]-2 \times F[I]+F[I-1]) \div (H \times 2)
** F''[I]-D2(H)=-(H*2)\times F''''(Z[I])\div 12, X[I-1]< Z[I]< X[I+1]
**
THIS MEANS THAT D2(H) = O(WHAT?)
      A \cdot H
      B \cdot H \star 2
CORRECT ANSWER IS: A,B ?
RIGHT
ASSUME THE FOLLOWING TABULATION FOR F(X)=(X*5)+(2\times X)
 I
                1
                          2
                                   3
           0
X[I]
       -.10000
                  0
                      .10000
                                .20000
                                          .30000
F[I]
     -.19999
                0
                      .20001
                                .40032
                                          .60243
FOR WHAT VALUE OF X DOES D2(H) NOT APPLY ?
Π:
      0 、
NO. TRY AGAIN OR TYPE 'HELP'.
FOR WHAT VALUE OF X DOES D2(H) NOT APPLY ?
\square:
      HELP
TO CALCULATE D2(H), YOU NEED VALUES F[I-1], F[I], F[I+1]. TRY AGAIN.
FOR WHAT VALUE OF X DOES D2(H) NOT APPLY ?
\square:
RIGHT. D2 CANNOT BE APPLIED AT THE OTHER ENDPOINT, EITHER.
SUPPOSE WE WISH TO APPROXIMATE F''(X[3]).
AT X[3], D2(H)=?
((.60243-2×.40032)+.20001)÷.01
RIGHT, D2(H) = .18.
THE EXACT VALUE IS F''(X[3])=.16. NOTE THAT THE ERROR FORMULA IS
      F''[I]-D2(H)=-(H*2)\times F'''(Z[I])\div 12, WHERE X[I-1]< Z[I]< X[I+1].
ON THE INTERVAL (X[2],X[4]),
MAX(ABS(F'''(X)))=?
Π:
      HELP
WE NEED TO FIND THE MAXIMUM OF THE FOURTH DERIVATIVE OF
      F(X)=(X*5)+(2\times X). TRY AGAIN.
MAX(ABS(F'''(X)))=?
Π:
```

Figure 2.5 Sample output from PARTC4



```
HELPMSG \leftarrow `F''''[I]-D2(H)=-(H \times 2) \times F'''''(Z[I]) \div 2. \quad AS \quad H \quad APPROACHES \quad 0,
LIM(F''''[I]-D2(H)) \div (H \times 2))=LIM(F'''''(Z[I]) \div 12)=F'''''(X[I]) \div 12, \quad WHICH \quad IS
                                                                                                                                                                                                                                                                                                                                                                                                                                                                  TELLMSG+("10+HELPMSG), 'D2(H) CANNOT
BE APPLIED AT X[0] SINCE F["1] IS NOT TABULATED, NOR AT X[4] SINCE F[5] IS NOT
                                                                                                                                                                                                                                                                                                                                                                                                                                             HELPMSC*'TO CALCULATE D2(H), YOU NEED VALUES F[I-1], F[I], F[I+1]. TRY AGAIN.
                                                                                                                                                                                                                                  TELLMSG \leftarrow (-10 + HELPMSG), 'SO D2(H) IS AN O(H + 2) APPROXIMATION TO F'''' (X[I]).
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            HELPMSG \sim USE THE FORMULA D2(H) = (F[I+1] - 2 \times F[I] + F[I-1]) \div (H \times 2) AT X[3].
                                                                                                                                                                                                                                                                                                                                                                                         RTANS+ 0.1,0.3
RTMSG+'RIGHT, D2 CANNOT BE APPLIED AT THE OTHER ENDPOINT, EITHER.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 =((,60243-(2×,40032))+,20001);(,01)=,18'
                                                                                                                                                                                                                                                                                                                ASSUME THE FOLLOWING TABULATION FOR F(X) = (X*5) + (2\times X)^{1}
                                                                                                                                                                                                                                                                                                                                                                 QUESTION←'FOR WHAY VALUE OF X DOES D2(H) NOT APPLY ?'
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               TELLMSG \leftarrow `NO, AT X[3], D2(H) = (F[4] - 2 \times F[3] + F[2]) \div (H \times 2)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    SUPPOSE WE WISH TO APPROXIMATE F''''(X[3]),
                                                                          QUESTION → CORRECT ANSWER IS; A,B ?
'THIS MEANS THAT D2(H)=O(WHAT?)'
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               QUESTION\leftarrow'AT X[3], D2(H)=?
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      RIMSG←'RIGHT, D2(H)=.18,'
                                                                                                                                                                                                          A CONSTANT. TRY AGAIN.'
                                                                                                                                                                                                                                                                                       →(FLG=1)/GETOUT
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              →(FLG=1)/GETOUT
                                                     H*2
                                                                                                                              RTMSG*'RIGHT'
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          RTANS+0.18
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    ASKNUMERIC
                                                                                                     RTANS \leftarrow 'B'
                                                                                                                                                                                                                                                                                                                                           TABLE3
[53]
                       [54]
                                                                             [55]
[56]
                                                                                                                             [57]
                                                                                                                                                                                                                                                            [60]
                                                                                                                                                                                                                                                                                   [61]
                                                                                                                                                                                                                                                                                                              [62]
                                                                                                                                                                                                                                                                                                                                         [63]
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                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            75]
```





-\*\*

$$D(h) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h}, \text{ was an approximation of } f'(x_i).$$

The other, 
$$p_2(h) = \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2}$$

approximated  $f''(x_i)$ . Each time the student typed "GO", the computer printed the values of h, D(h), and D2(h) for approximations at  $x_i = 0$ . On each iteration the previous value of h was multiplied by .1. Thus the student was able to observe the convergence of D(h) and D2(h) to the true value of 1 as the interval size h decreased. After a number of iterations D(h) and D2(h) began to grow more inaccurate because of the roundoff error. The student had complete control of the number of iterations performed.

A similar but more general mechanism followed. In this case the student was allowed to choose from a list of four functions, that function whose first and second derivatives were to be approximated. The four functions available were

- (a)  $f(x) = e^{x}$
- (b)  $f(x) = x^5 + 2x$
- (c)  $f(x) = \sin x$
- (d)  $f(x) = 2(\sin x x\cos x)$ .

A fifth choice, labeled "NONE", permitted the student to terminate this part of the lesson. After choosing a function, the student was asked to choose the point,  $\mathbf{x_i}$ , at which he wished to approximate the derivatives. The value of  $\mathbf{x_i}$  could be any rational number. For each function and point so chosen, the student could select any



number of values of h, the interval size. For each h specified, the corresponding values of D(h) and D2(h) were printed. The student was free to choose any interval sizes, perhaps strictly decreasing, perhaps not. It was hoped that students would explore the effect of roundoff error for various interval sizes at various approximation points. When the student typed "ENUF", he was asked if he wanted to choose another x<sub>i</sub> for the same function. If so, he was given that opportunity and could then specify new values of h. If not, he was presented the function list again, from which he could choose another function or "NONE". Figure 2.7 shows one possible use of the features of PARTD.

# 2.4.4 Data Capturing

The functions ASKMC and ASKNUMERIC described in Section 2.4.3.2 recorded for each question whether HELP was requested and whether the student's answer was right or wrong. A question was counted as answered incorrectly only if the student never gave the correct response. Thus an incorrect response, followed by a request for HELP, followed by a correct response, was counted as being correct. These data were not kept separately for each question, but were added within each lesson section and recorded at the end of the section. Moreover, the time at which the student started the section and the elapsed time spent on that section were recorded.



```
SIZE H. FOR EACH VALUE OF H SPECIFIED, THE VALUES OF H, D(H), AND D2(H) WILL BE PRINTED. TYPE 'ENUE' WHEN YOU ARE READY TO CHANGE X[I] OR THE FUNCTION.
THERE FOLLOW FOUR FUNCTIONS. YOU MAY CHOOSE THE FUNCTION WHOSE DERIVATIVES
                     YOU WISH TO APPROXIMATE, THE POINT OF APPROXIMATION X[1], AND THE INTERVAL
                                                                                                                                                                                                                                                                                                                                                                                                      D2(H)=2.718282055
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          D2(H)=2.718272274
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   D2(H)=2.642330799
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    D2(H) = -4444.0892099
                                                                                                                                                                                                                                                                                                               X[I]=1
                                                                                                                                                                                                CHOOSE A FUNCTION: A,B,C,D,E?
                                                                                                                                                    F(X)=2\times(SIN(X)-X\times COS(X))
                                                                                                                                                                                                                                                                                                                                                                                                   D(H)=2,718282282
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        D(H)=2.718281828
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                D(H)=2.718281825
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   D(H)=2.7182816
                                                                                                                                                                                                                                              CHOOSE A VILUE FOR X[I],
                                                                                                           F(X)=(X*5)+(2\times X)
                                                                                                                                F(X) = SIN(X)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                .000000000
                                                                                      F(X)=E\star X
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         .0000001
                                                                                                                                                                                                                                                                                                                                                                                                                                                                     .00001
                                                                                                                                                                                                                                                                                                                                                                             .001
                                                                                                                                                                            NONE
                                                                                                                                                                                                                                                                                                              F(X) = E * X
                                                                                                                                                                                                                                                                                                                                                                                                   H=0.001
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      H=1E^{-}5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               H=1E^{-}9
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            H=1E_7
                                                                                                                                                                                                                                                                                                                                  ?=H
                                                                                                         В.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             :=H
```

Figure 2,7 Sample output from PARTD



Each student started the lesson by executing the function BEGIN, which recorded some information, introduced the lesson, and gave directions regarding the printed output. Based upon the student's responses to several questions, the function recorded the student's location, the time of day, whether the student had used a computer terminal before, and whether he was familiar with APL.

In the original PICLS program, this data collection was done by the system. In the APL\360 version used in the experiment, the data could be collected by the program but would have been lost if the system crashed or when the next lesson section was brought into the active workspace. mentioned previously, there are APL\360 system commands which allow the user to load, copy, and save data but they cannot be included in a function definition and executed later. It was therefore necessary to have the student execute the proper commands at the end of each section. majority of the students did not know any APL and thus did not recognize failure of the system to perform the requested action. For this reason, some of the desired data were never recorded. Other data cannot be considered accurate because of untimely system failures. For example, if a student had progressed through a lesson section entering some correct and some incorrect responses, with an occasional request for help, and the system crashed just before he saved the data, there were two alternative courses of action upon



### system restoration:

- (1) he could restart the section, in which case the response and time that eventually saved would not accurately reflect his <u>first</u> pass through the section, or
- (2) he could start the next section, in which case no data would be saved.

In a small-production, one-time session such as this, the data could be reconstructed through inspection of the output, but such a course is not practical in programs of larger scale. It is the author's feeling that as much data collection as possible should be done automatically at the time of data generation.

## 2.5 Equipment

Two very similar types of terminals were used for the experiment, the Datel Thirty-21 and the IBM 2741. Moreover, two terminals of each type were used. Both types of terminals look like electric typewriters with slight modifications. The differences between the two types apparent to the user are listed in Table 2.8.

#### Datel Thirty-21

Rests on table like a typewriter

Electronic package attached at rear of typewriter

Three status lights

Outer cover blue-green

### IBM 2741

Mounted in special table
Electronics mounted under table
No status indicators
Outer cover gray



Table 2.8 Differences between terminals

The only difference which seems to have any potential influence upon the user is the absence of any indicators on the IBM 2741 as to the status of the terminal. The Datel terminals have three lights, labeled "READY", "PROCEED", and "CHECK". The READY light indicates that a connection exists between the terminal and the computer. PROCEED indicates that the keyboard is unlocked and the user can enter data. The CHECK light is illuminated when a parity error is found in transmissions to the terminal. When an error occurs the keyboard is locked and the CHECK light illuminated. There is a key, not found on the IBM 2741, which is used to free the terminal from this check status.

An attempt was made to use the Datel terminals exclusively, but scheduling difficulties necessitated use of the two 2741's. In general, students who did not finish the lesson in one session returned to the same location for all following sessions. The single exception completed 70% of the lesson at Location 1 and 30% at Location 2. For data analysis, he is considered to have worked at Location 1 exclusively. As can be seen in Table 2.9, the majority of students used Terminal 1. Three of the four locations were judged to be not significantly different in physical conditions (i.e., lighting, heat, space, solitude). Location 3 was a small room which contained two other terminals. It is quite possible that noise from the other terminals and their users proved distracting to the subjects.



As explained earlier, Treatment Groups 1 and 2 operated the terminal in the normal manner. The paper fed continuously through the terminal and fell to the floor behind the table. A student could, at any time, pull the paper back and look at any part he wanted to see. Figure 2.10 shows the terminal as it appeared to these students.

The objective with Group 3 was to simulate a CRT in terms of access to the output. A box-like device was constructed to fit on the terminal so that the printed output passed through the terminal and into a slot in the box, allowing the user to see only the most recent part of the output. The effect is similar to using a CRT with "scrolling" (i.e., displaying new data at the bottom of the screen and moving previous display up one line). The terminal as it appeared to Group 3 is shown in Figure 2.11.

Location			1	2	3	4
Terminal			Datel Thirty-21	Datel Thirty-21	IBM 2741	IBM 2741
Subjects,	Group	1	4	2	2	1
Subjects,	Group	2	3	2	1 .	0
Subjects,	Group	3	7	0	0	1
Subjects,	Total		14	4	3 '	2

Table 2.9 Use of terminals



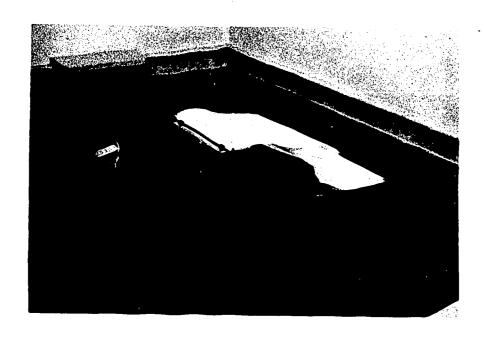


Figure 2.10 Terminal as used by Group 1 and Group 2

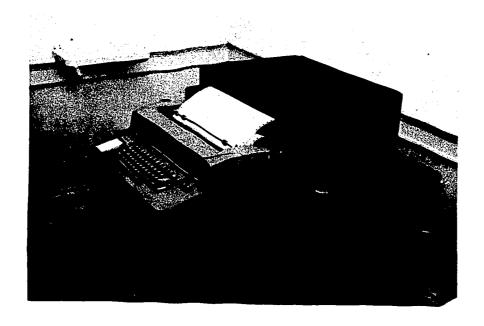


Figure 2.11 Terminal as used by Group 3



The amount of information displayed at one time is important to the CRT user. In this particular experiment it seemed that if members of Group 3 were indeed handicapped by the limited access to the paper that handicap would become more severe as the amount of output available was made smaller. It was therefore decided that the amount of output accessible should be an upper bound of the amount actually shown on a CRT. A survey of CRT screen dimensions [1] led to a choice of 40 lines with 80 characters per line. While each of these dimensions is provided on some CRT, no model known to the author provides both. The number of lines visible was limited by the distance from the terminal's type element to the slot in the appended box. The maximum number of characters per line was limited by the author in programming the lesson.

# 2.6 Measurement of Treatment Effect

The subjects were given a 30 minute pretest on the first day of the experiment. The pretest was explained to the student as a means of measuring their current knowledge of numerical differentiation so that the teaching program could be evaluated for effectiveness. No mention was made of the three treatment groups. In constructing the test, a distinction was made between questions designed to measure the student's mathematical background and those designed to measure the student's specific knowlege of numerical differentiation. The former will hereafter be referred to



collectively as Pretest 1, the latter, Pretest 2. The intention was to perform analyses of covariance using each of the pretests as covariate to determine the effect of the various treatments. On the actual test, a copy of which is Appendix C, questions numbered 1, 3, 5, 6 formed Pretest 1; questions numbered 2, 4, 7 formed Pretest 2. A moderate positive correlation was found to exist between the scores on each pretest and those on the posttest and between the scores on the two pretests, as shown in Table 2.12.

	Pretest 2	Posttest
Pretest 1	,4224	。3264
Pretest 2	وي وهذا ۱۹۵۸ منځ بون	. 4681

Table 2.12 Coefficients of correlation between scores on pairs of tests

The questions on the posttest, which was to be completed within 50 minutes, were based directly upon the presentation made to the students at the terminal. All questions contributed equally to the total score. After the posttest was given, a questionnaire on which the students were asked to record their reactions to the lesson was distributed. Copies of the posttest and questionnaire can be found in Appendices D and E, respectively.

Construction and scoring of the tests, as well as construction of the questionnaire, were done by the author. The students' names on the papers were replaced by numbers by an impartial observer before the papers were graded, thereby



eliminating the grader's bias. Independent numbering schemes were used for the pretest and posttest to ensure the absence of grader prejudice.

## 2.7 Other Design Features

# 2.7.1 Proctors

Each student was observed during his terminal session by a graduate student from the Department of Computer and Information Science. These proctors were present to ensure that the experimental conditions were adhered to and to assist the student with system difficulties. In particular, they placed the box on the terminal, collected output, assisted with initial sign-on, ensured that the data-storing and program-loading system commands were properly executed, assisted the student with communications failures, and recorded any observations they felt significant.

Although the presence of a proctor was desirable in this controlled experiment, it may not be the best arrangement.

On the questionnaire, 15% of the students responded that they were "slightly bothered" by the proctor's presence.

### 2.7.2 Realistic Setting

The experiment was conducted in an active academic environment, not an artifical, experimental situation. This
"real world" approach to the study created some uncontrolled
variables such as the amount of use the students made of the



textbook, the amount of study time per student, and the possibility of members of Groups 2 and 3 borrowing output from members of Group 1. However, it was felt that for any results to be applicable to a realistic problem, they must be obtained in a realistic situation. In keeping with this philosophy, the students were referred to Conte [3] for ore-lesson reading. Those who did not have a copy of the session output were at liberty to study the same or any other text after the lesson.

# 2.7.3 Placebo Effect

In experiments of this nature it is sometimes the case that one group performs better than others because they think they are receiving the superior treatment. In order to reduce this placebo effect it is necessary to keep the treatments unknown to the subjects. The proctors made every attempt to have the terminal attachment in place before members of Group 3 arrived and to remove it before other subjects entered the room. The instructions for disposing of the output were printed by the program, based upon previously stored data on group membership. It is not possible to determine how much information was relayed between groups in informal discussions.

# 2.7.4 Time Span

The pretest was given on a Wednesday, at which time the students chose the times for their terminal sessions. Approximately 22% of the sessions were held on the following two



days, with the remainder being conducted on Monday, Tuesday, and Wednesday of the following week. The posttest was given nine days after the pretest, with the time lapse between lesson and posttest ranging from two to eight days.

## 2.7.5 Group Assignment

Students were assigned to treatment groups using the APL\360 function "deal". A vector was generated, each component of which was a random number from one to three. The alphabetic class roll was matched with this vector to assign treatments. When the absence of three students from the pretest produced unbalanced groups, a number <u>n</u> was chosen at random from the integers 1 through 7. The alphabetically nth member of Group 2 was changed to Group 3.



## Chapter III

#### THE RESULTS

## 3.1 Comparison of Treatment Groups

## 3.1.1 Posttest Scores

An analysis of covariance between the three treatment groups was performed using the two pretest scores as covariates and the posttest scores as the dependent variable. The results of the analysis are summarized in Table 3.1. There is no evidence to support rejection of the null hypothesis that the three treatment group means are equal when pretest differences are controlled.

df	Sum of Squares (y)	Sum of Squares (within)	Sum of Squares (between)	df	Mean Square
2	217.375	202,254	15.121	2	7.560
17	1757.188	270.140	1487.047	15	99.137
19	1974.563	472.395	1502.168	17	وف ينتث
	2 17	Squares (y) 2 217.375 17 1757.188	df     Squares (y)     Squares (within)       2     217.375     202.254       17     1757.188     270.149	df     Squares (y)     Squares (within)     Squares (between)       2     217.375     202.254     15.121       17     1757.188     270.140     1487.047	df         Squares (y)         Squares (within) (between)         df           2         217.375         202.254         15.121         2           17         1757.188         270.149         1487.047         15

F = .076, not significant

Table 3.1 Analysis of covariance of posttest scores using pretest scores as covariates.

Table 3.2 shows the mean score for each group on each measure. Notice that Group 2 scored consistently lower on all tests, perhaps for reasons cited in Section 2.2.1. A complete tabulation of test scores is found in Appendix F.



	Pretest 1	Precest 2	Posttest
Group 1	42.57	33.86	86.50
Group 2	31.33	24.50	77.17
Group 3	39.57	31.43	82.88

Table 3.2 Test score means by treatment groups

Source	Sum of Squares	<u>df</u>	Square	F-Ratio	р
Question 1 Treatment	191.93-	2	95.97	3.60	p> .05
Error	479.88	18	26.66		p / 100
Question 2					
Treatment	46.91	2	23.45	.72	иs
Error	585.76	18	32.54		
Question 3					
Treatment	11.03	2	5.51	.79	NS
Error	125.92	18	6.60		
Question 4					
Treatment	16.60	2	8 <b>.30</b>	1.01	NS
Error	148.36	18	8.24		
Question 5					
Treatment	6.40	2	3.20	.19	NS
Error	298.55	18	16.59		

Table 3.3 Analysis of variance between treatment groups by posttest question.

A simple analysis of variance was performed to determine whether the group scored significantly differently on individual posttest questions. The results are summarized in Table 3.3. Notice that no significant difference was found except on Question 1, which reads, "Explain what it means for D(h) to be an O(h<sup>k</sup>) approximation to a number <u>a</u>." With the question being graded on a 20-point scale, the mean scores were



as follows:

Group 1 14.375

Group 2 8.000

Group 3 15.000

The significance of the F value seems attributable to the low scores of Group 2. If the low scores were due to lack of hard copy to study after the session, one would expect low scores from Group 3 as well. In the absence of such low scores, one suspects that the difference is not due to the treatment. Accordingly, an analysis of covariance was performed using the two pretest scores as multiple covariates and the scores on Question 1 as dependent variable. The result, F (2,15) = 1.802, is well within the range of non-significance at the .05 level.

## 3.1.2 Lesson Performance

In all instruction, and especially in CAI, a desirable goal is to achieve maximum learning in a minimum amount of time. Thus if two methods of teaching produce the same results, but one method does so in significantly less time, then it is judged to be more desirable. With this in mind, an analysis of variance was performed on the total working time of the three groups. The times used were adjusted to exclude time consumed by system failures. Although the difference was not significant, it is interesting to note that Group 3 required an average of 10 minutes less than



any other group.

During the terminal session, Groups 1 and 2 had the same access to the printed output while Group 3 was restricted. It therefore seems reasonable to compare Group 3 with the union of Group 1 and Group 2 to determine the effect of limiting access to the output. One notes from Table 3.4 that no difference was observed in the number of requests for help or in the number of wrong responses. (A wrong response is here defined as a question to which the student did not give the correct answer in the maximum allowable number of tries.) Again, there is an apparent, although statistically insignificant, difference in completion times. The relatively high number of wrong responses and requests for help by Group 2 are consistent with that group's relatively poor scores on all three tests.

	Group 1	Group 2	Group 3	Group 1 and 2
Hean completion time in minutes	1.24	129	114	126
Hean number of wrong responses	5.3	7.4	<b>6.7</b>	6.4
Mean number of requests for help	3.7	5,1	4.9	4.4

Table 3.4 Group means for completion times, wrong responses, and requests for help

## 3.1.3 Questionnaire Responses

In determining the use made of the output while at the terminal, Group 3 should be compared with the union of Group



1 and Group 2. Of the six members of Group 3 who returned questionnaires, only two were more than "slightly frustrated" at not being able to see the output. On the other hand, 60% of Groups 1 and 2 said they often referred to the portion of the output that would have been hidden from Group 3. This suggests that having the output may be useful, but that Group 3, having never developed the habit of looking at the output, did not miss it. Indeed, the ability to look back was judged "considerably useful" or "very useful" by 87% of Groups 1 and 2.

With regard to output use after the session, Groups 2 and 3 may be combined. Perhaps the most significant point to arise from the questionnaire is that the students who had no session output studied the textbook instead. The total amount of study time was approximately the same for the two conditions but Group 1 studied the session output almost exclusively. Of this group, 70% ranked the output more useful for later study than the textbook.

Because of the printing method employed in most typewriter terminals, a linear notation must be used to achieve reasonable efficiency. Combined with a lack of lower case letters, this causes the formula  $D(h) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h}$  to be printed as  $D(H) = (F(X[I+1]) - F(X[I-1])) \div (2 \times H)$ . All three treatment groups reported experiencing "moderate" to "considerable" difficulty reading such notation.



## 3.2 Other Comparisons

In addition to comparisons of treatment groups, several questions arise which are interesting in their own right and the answers to which serve as validation for the experiment. The results of these comparisons, all of which are simple analyses of variance among posttest scores, are summarized in Table 3.5 and explained below.

Basis for Grouping	<u>df</u>	Ŗ	Critical F	Experimental F
Day of terminal session	1,19	NS	4.38	3.53
Terminal location	3,17	NS	3.20	2.61
Type of terminal	1,19	NS	4.38	1.60
Experience with terminals	1,18	NS	4.41	0.16

Table 3.5 Degrees of freedom, significance thresholds, and F ratios for miscellaneous comparisons of post-test scores

The terminal sessions ranged over a seven day period, with the posttest coming two days after the last session. It seemed possible that those students who had completed the lesson most recently would do better on the posttest because of the time difference. The students were divided into two groups  $(n_1 = 10, n_2 = 11)$  according to the day of their terminal session. These two chronological groups were compared on the basis of posttest scores. There was no significant difference in scores. Moreover, the difference that did exist, seven points on a 100-point scale, was in favor



of the chronologically first group. Since the students chose their own session times, this result may reflect a tendency of the poorer students to postpone the session as much as possible.

Four terminals, in four locations, were used for the experiment. Two of these were Datel Thirty-21's and two were IBM 2741's. Differences in external conditions such as lighting, noise, temperature, and other nearby activity have potential influence upon the students' retention of subject matter. The appearance and functions of the two types of terminals may possess similar potential. Analyses of variance of posttest scores using both terminal type and terminal location as grouping criteria were performed. In neither case was a significant difference found.

Finally, a comparison was made to determine whether students who had never used a terminal before suffered from a form of "stage fright". Although they may have felt ill-at-ease during the lesson, these first-time users did not score differently on the posttest.

### 3.3 Summary

## 3.3.1 Efficiency of Learning

Neither the use of printed output during the terminal session nor its availability after the session had a significant effect on posttest scores when college students were taught numerical differentiation through computer-assisted



instruction. Those students who had access to the paper during the session took 10% more time to complete the lesson, an observable but statistically insignificant difference.

Access to the output was not found to affect the correctness of responses or the number of times the students requested clues during the lesson.

# 3.3.2 Student Attitudes

Those students who had access to the output during the session felt that it was very useful, while those who had no access were not particularly bothered by that fact. Those who received the paper after the terminal session used it extensively as a study guide. Those not receiving the output reported that they would have studied it, had it been available. Nearly all of the students without the printed output compensated for its absence by studying related material in the textbook.



### Chapter IV

### THE CONCLUSIONS

## 4.1 Apparent Implication

Under the conditions of this experiment and subject to the limitations discussed in the following section, it appears that no ill effects would be suffered from use of a terminal which does not produce hard copy output. This is equally true when the quality of learning is measured by the correctness of student responses during the session and when it is measured by posttest scores.

If hard copy were universally insignificant then for those people who were about to establish, expand, or upgrade a CAI system, the CRT would become clearly more desirable than the typewriter terminal. One could then have the speed and flexibility of the CRT without having to pay for a separate printer or penalize the student by not providing hard copy. For those already committed to CRT's, programming complexity would be reduced by elimination of the need for making provisions for hard copy.

Students with access to the printed output during the terminal session (Groups 1 and 2) took slightly longer to complete the lesson. This difference may be a result of the students looking back at previous output which was



not available to Group 3. The fact that Group 3 learned as much in less time seems to indicate that the extra time was not used effectively.

The most obvious use of hard copy output is as a reference for further study. As would be expected, those students who were allowed to take the output with them after the session used it extensively as a study guide. Those not receiving the output reported that they would have studied it, had it been available. Although this post-lesson study may not occur in all CAI situations, it was certainly not unexpected in this particular instance.

It is noteworthy, in the author's opinion, that those students who did not receive the output spent as much time in post-lesson study as the group receiving the output, compensating for the lack of hard copy by studying the text-book. Certainly, many factors in the experimental design, notably the availability of a substitute source (the text-book) and the presumed high motivation of the students, contributed to this phenomenon. Nevertheless, there is evidence here that students can and will compensate for the lack of hard copy. It may even be the case that this forced exposure to two different sources contributes to a broader understanding of the subject matter.

Difficulties in reading the linear notation imposed by the typewriter were equally severe for all treatment groups. The problem was especially acute in this instance



because it was the students' first (and only) lesson using that notation. Although students may experience less difficulty as their exposure to linear notation increases, the notation remains inconsistent with standard mathematical usage and therefore incommodious.

# 4.2 <u>Limitation on Generalization</u>

Certain limitations are imposed upon the generality of any experimental results by the circumstances under which the study was made. The experimental design and the rationale thereof are discussed in Chapter III; this section considers the applicability of the results of this study to other situations.

The value of hard copy output is certainly dependent upon the learning objectives established by, or in conjunction with, the author of the CAI program. The objectives, of course, determine the content of the lesson and the desired detail of recall. The desired outcomes of the lesson used in this study were understanding of concepts and ability to apply formulas to specific data. It seems reasonable that hard copy would be more valuable as a study guide if specific formulas were to be memorized or if one were learning, for instance, state capitals.

The tests used in this investigation reflect the objectives of the lesson. Each of the five questions on the posttest had a value of twenty points. Partial credit was



given for partially correct answers. It is possible, although unlikely in this case, that the type of test affects the variance among treatment groups.

Also related to the desired outcome of instruction is the mode of instruction used. The role of hard copy output may be quite different in simulation and student-controlled problem-solving modes from the role in author-controlled mode. No attempt has been made to measure the value of hard copy other than in computer-administered programmed instruction; no attempt should be made to generalize these results to any other mode.

It is worth noting here that the students who received no hard copy output to take away from the terminal did have access to a source of similar information, the textbook. When the computer is used in modes other than author-controlled, there may not be such a comparable source, in which case the value of hard copy output may well increase.

The study reported herein is based upon one lesson and a test that followed within ten days. It is not difficult to imagine that a copy of the output may become more valuable as a study guide as the time between lesson and test increases. It is not clear how students' use of hard copy output would change if a series of lessons were used instead of just one.

Perhaps the most important factor one must consider in generalizing from these results is the characteristics of



the students. The subjects used in this investigation were of above average intelligence, well motivated, mature, and well accustomed to the academic environment. When confronted with the lack of hard copy output they had the ability and the initiative to compensate. Obviously, one would not expect all levels of students to react in the same manner.

To the author's knowledge, this is the only investigation that has been made to determine the value of hard copy output. It should be regarded as a study of one particular CAI situation; generalization should be made only in the light of the preceding discussion.

Further investigations should proceed along several lines in an attempt to determine the circumstances under which hard copy output is beneficial to the student. The most obvious area of interest is that of student characteristics, particularly age. Other modes of computer use should be investigated as well as other subject matter. To ensure validity, future studies should use larger treatment groups for extended periods of time.



#### **BIBLIOGRAPHY**

- 1. Auerbach Corporation. Auerbach Data Communications Reports. 3 vols. Philadelphia, 1966.
- 2. Carpenter, C. R. and Greenhill, L. P. Comparative

  Research on Methods and Media for Presenting Programed Courses in Mathematics and English. University Park, Pa.: The Pennsylvania State University, 1963.
- 3. Conte, S. D. <u>Elementary Numerical Analysis</u>. New York: McGraw-Hill, 1965.
- 4. Feldhusen, John and Szabo, Michael. "The Advent of the Educational Heart Transplant, Computer Assisted Instruction: A Brief Review of Research." Contemporary Education, XL, No. 5 (April, 1969).
- 5. Gerard, R. W. (ed.). Computers and Education: A Work-shop Conference at the University of California, Irvine. New York: McGraw-Hill, 1967.
- 6. Glaser, Robert et al. The Interface between Student and Subject Matter. Pittsburgh: Learning Research and Development Center, University of Pittsburgh, 1966.
- 7. International Business Machines Corporation. APL\360
  User's Manual. Form H20-0683, 1969.
- 8. Johnson, D. W. and Borman, K. G. "Relative Effectiveness of Various Modes of Stimulus Presentation through CAI."

  Experimentation with Computer-Assisted Instruction in Technical Education. University Park, Pa.: The Pennsylvania State University, 1967, 27-40.
- 9. Keene, Jeness. "Brentwood Revisited: CAI's Two-Year Trial," Nation's Schools, MXXXII, No. 4 (October, 1968).
- 10. Oldehoeft, Arthur E. Computer-Assisted Instruction in Teaching Numerical Methods (A Computer System to Teach Computational Mathematics). Layfayette, Indiana: Purdue University, 1970.



- 11. Roscoe, John T. <u>Fundamental Research Statistics for the Behavioral Sciences</u>. New York: Holt, Rinehart and Winston, 1969.
- 12. Suppes, Patrick. "The Uses of Computers in Education," <u>Scientific American</u>, CCXV, No. 3 (September, 1966).
- 13. Suppes, Patrick, "How Far Have We Come? What's Just Ahead?," Nation's Schools, MXXXII, No. 4 (October, 1968).
- 14. Swets, John A. et al. Learning to Identify Nonverbal

  Sounds: An Application of a Computer as a

  Teaching Machine. Cambridge, Massachusetts:

  Bolt, Beranek and Newman, 1962.
- 15. Swets, John A. et al. "Computer-Aided Instruction in Perceptual Identification," Behavioral Science, XI, No. 2 (March, 1966).
- 16. Wodtke, Kenneth H. and Gilman, David A. "Some Comments on the Efficiency of the Typewriter Interface in Computer-Assisted Instruction at the High School and College Levels." (Paper presented at the Annual Convention of the Association of Educational Data Systems, Philadelphia, May 1966).



#### APPENDIX A

#### LESSON OUTLINE

- I. PARTA Order of an approximation
  - A. Define D(h) to be "order h" if  $\lim_{h \to 0} \left( \frac{A D(h)}{H} \right) = \text{a nonzero constant.}$
  - B. From Taylor's expansion of  $f(x_0 + h)$  about  $x_0$ , show that  $D(h) = \frac{f(x_0 + h) f(x_0)}{h}$  is of order h.
  - C. From Taylor's expansions of  $f(x_0 + h)$  and  $f(x_0 h)$ about  $x_0$ , show that  $D(h) = \frac{f(x_0 + h) f(x_0 h)}{2h}$  is of order  $h^2$ .
- II. PARTB Functions tabulated on an equally spaced set of points
  - A. Introduce tabular format; note equal intervals.
  - B. Apply the two approximations developed in PARTA to one point in a given table.
- III. PARTCL O(h) approximation to f'(x)
  - A. Given a table and the analytic form of the function, find the maximum error.
  - B. Apply the O(h) approximation formula at three different points.
  - C. Compare each approximation with the true value.
  - D. Compare each error with the maximum error.

- IV. PARTC2 O(h<sup>2</sup>) approximation to f'(x)
  - A. Note that  $D(h) = \frac{f(x_{i+1}) f(x_{i-1})}{2h}$  cannot be used at the ends of a tabulated interval.
  - B. Find the maximum error for this formula in the given interval.
  - C. Apply the formula at the same points used in PARTC1.
  - D. Compare the O(h), O(h<sup>2</sup>) and true values at the three points.
- V. PARTC3  $O(h^2)$  approximation to  $f'(x_0)$  and  $f'(x_N)$ 
  - A. From Taylor's expansions of  $f(x_0)$ ,  $f(x_1)$  and  $f(x_2)$  about  $x_0$ , derive

$$D(h) = \frac{-3f(x_0) + 4f(x_1) - f(x_2)}{2h}$$

- B. Find the maximum error in the interval.
- C. Apply the formula and note that the error falls within the predicted bounds.
- D. Do the same sequence for  $f'(x_N)$ .
- VI. PARTC4  $0(h^2)$  approximation to f"(x<sub>i</sub>), 1 < i < N
  - A. From Taylor's expansions of  $f(x_i)$ ,  $f(x_{i+1})$  and  $f(x_{i-1})$ , derive

D2(h) = 
$$\frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2}$$
.

- B. Find the maximum error in the interval.
- C. Apply the formula and note that the error falls within the predicted bounds.

- VII. PART D Computational accuracy of numerical differentiation
  - A. Using  $D(h) = \frac{f(x_{i+1}) f(x_{i-1})}{2h}$  and  $D2(h) = \frac{f(x_{i+1}) 2f(x_{i}) + f(x_{i-1})}{h^2}$ , show that
    - error approaches zero as h approaches zero.
  - B. Show roundoff error in approximations to f'(x) and f''(x) for  $f(x) = e^{x}$  by letting h approach zero.
  - C. Present student with functions and mechanism for observing roundoff error under student control.

#### APPENDIX B

## FUNCTIONS ASKMC AND ASKNUMERIC

∇ ASKNUMERIC

```
[1]
       COUNT+FLG+HLPFLG+0
[2] QU:QUESTION
      \rightarrow (HELPX,STOPX,((\rho_RTANS)\rho_RTX),RONGX)[(HELP,STOP,RTANS)\tauANS\-\|]
[3]
[4] HELPX: \rightarrow QU, (\rho \Box \leftarrow HELPMSG), HLPFLG \leftarrow 1
[5] STOPX: \rightarrow (FLG \leftarrow 1)/0
[6] RTX: \rightarrow 0, (\rho \square \leftarrow RTMSG), RTREC \leftarrow RTREC + 1, HLPFLG
[7] RONGX COUNT + COUNT + 1
[8]
      →(COUNT≥2)/ENDX
[9]
      →QU,ρ□←RONGMSG
[10] ENDX: TELLMSG
[11] RONGREC+RINGREC+1, HLPFLG
[12] PAUSE
      ∇ASKMC NUM
[1]
      COUNT+3-NUM
[2]
      HLPFLG←FLG←0
[3] QU:QUESTION
[4]
       →(4=ρANS←①)/FOURLETS
[5]
        \rightarrow CKX
[6] FOURLETS: \rightarrow (FLG \leftarrow ANS \land = "STOP")/0
[7]
      \rightarrow (ANS \land . = 'HELP')/HELPX
[8] INVAL:→QU, p ← 'YOU MUST CHOOSE ONE OF THE GIVEN ANSWERS OR
          TYPE ''HELP''.'
[9] HELPX:\rightarrow QU, (\rho\Box\leftarrow HELPMSG), HLPFLG\leftarrow 1
[10] CKX:\rightarrow (1 \neq \rho ANS)/INVAL
[11] COUNT←COUNT+1
[12] \rightarrow (ANS=RTANS)/RTX
[13] \rightarrow (COUNT \ge 2) / ENDX
[14] \rightarrow QU, \rho \square \leftarrow RONGMSG
[15] RTX:\rightarrow 0, (\rho\Box\leftarrow RTMSG), RTREC\leftarrow RTREC+1, HLPFLG
[16] ENDX:TELLMSG
[17] RONGREC+RONGREC+1, HLPFLG
[18] PAUSE
```



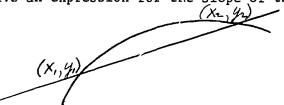
### APPENDIX C

#### PRE-EXAMINATION

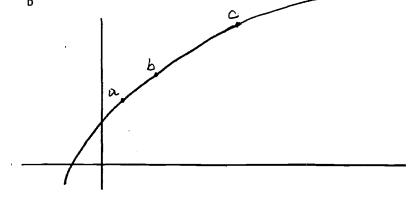
**COMP 150** 

October 21, 1970

1. Give an expression for the slope of the line shown below.



- 2. Explain the relationship between discretization error and roundoff error.
- 3. Given  $f(x) = 2x^5 7x^3 + 4$ , evaluate  $\lim_{a\to 0} \left(\frac{1^2}{5} f'''(2+a)\right)$ .
- 4. Given a tabulated set of values for  $x_i$  and  $f(x_i)$ , i=1,2,...,8, why can  $D(h) = \frac{f_{i+1} f_{i-1}}{2h}$  not be applied at i=8?
- 5. What is the geometric significance of the first derivative of a function ?
- 6. State Taylor's formula with remainder for expansion of f(x) about a point, a.
- 7. Suppose you are given the graph below and the coordinates of the points a,b,c,d. How would you estimate the value of the derivative of f evaluated at  $x_b$ ?



#### APPENDIX D

### POST-EXAMINATION

COMP 150

October 30, 1970

#### NUMERICAL DIFFERENTIATION

- 1. Explain what it means for D(h) to be an  $O(h^k)$  approximation to a number a.
- 2. Given the Taylor expansions

$$\begin{split} f_{i+1} &= f_i + hf_i' + \frac{h^2}{2}f_i'' + \frac{h^3}{6}f^{i'}''(w) \quad \text{where} \quad x_i < w < x_{i+1} \\ f_{i-1} &= f_i - hf_i' + \frac{h^2}{2}f_i'' - \frac{h^3}{6}f^{i'}''(v) \quad \text{where} \quad x_{i-1} < v < x_i \\ \text{derive the formula for the error in the approximation } D(h) &= \frac{f_{i+1} - f_{i-1}}{2h} \\ \text{Of what order is this approximation} \end{split}$$

3. The following are all approximations to  $f_i$ . What are the advantages and disadvantages of each? That is, under what circumstances would each be used?

(a) 
$$D(h) = \frac{f_{i+1} - f_i}{h}$$

(b) 
$$D(h) = \frac{f_{i+1} - f_{i-1}}{2h}$$

(c) 
$$D(h) = \frac{-3f_i + 4f_{i+1} - f_{i+2}}{2h}$$

4. Given the following table, apply at  $x_3$  each of these approximations.

$$D(h) = \frac{f_{i+1} - f_{i-1}}{2h}$$

$$D(h) = \frac{f_{i-1} - 2f_{i} + f_{i+1}}{h^{2}}$$

5. In the context of numerical differentiation, explain what is meant by the "optimal value of h".



### APPENDIX E POST-EXAMINATION QUESTIONNAIRE

COMP 150 Computer-Assisted Instruction Project

A	t	tı	t	ud:	ì	na	1	Su	r	V	e	У	•
---	---	----	---	-----	---	----	---	----	---	---	---	---	---

October 30, 1970

The answers to the following questions will be used in conjunction with your test scores in attempting to draw some conclusions regarding the value of the printed output in computer-assisted instruction. While inclusion of your name would be helpful in comparing reactions to performance, you should feel free to return this form unsigned if you prefer.

- A. 1. How many semester hours of mathematics have you taken?
  - 2. Circle the best estimate of your cumulative math grades.

A B C D F

- 3. What is your major field?
- 4. Circle your classification.
  - Jr. Sr. Grad 1 Grad 2 Grad 3 Other
- B, Please circle the word or group of words which best describes your leelings or actions.
  - 1. Before the terminal session, I studied the appropriate section of Conte

none less than 15 min. 15-30 min. 30-60 min. more than 60 min.

2. After the terminal session, I studied the appropriate section of Conte

none less than 15 min. 15-30 min. 30-60 min. more than 60 min.

3. I had trouble reading the linear notation (subscripts, exponents, fractions).

not at all slightly moderately considerably very much

4. Difficulties with the terminal, telephone, and computer interfered with my concentration on the material presented.

not at all slightly moderately considerably very much

5. The presence of the proctor bothered me.

not at all slightly moderately considerably very much

6. The clues presented when I typed "HELP" were actually helpful in understanding the material.

never seldom occasionally usually almost always Never typed "HELP"



- 7. I purposely typed a wrong answer just to see what would happen.

  never seldom occasionally usually almost always
- 8. I asked for help when I did not really need it.

never seldom occasionally usually almost always

9. Trying to put mathematical expressions in the proper format interfered with my concentration on the material presented.

not at all slightly moderately considerably very much didnot enter expressions

There were three different conditions in the experiment described below as C, D, E. Circle the letter corresponding to the group you were in and answer the associated questions.

- C. A box-like device on the terminal limited my access to the paper.
  - 1. I felt frustrated at not being able to see the previous output.

    not at all slightly moderately considerably very much
  - If I had been allowed to take the paper along, I would have studied it later.

no maybe probably surely

3. I borrowed some one else's paper to study.

yes no

4. During the terminal session, I pulled the paper out of the slot in order to read something.

never seldom often

- D. I could look at the paper during the terminal session, but was not allowed to take it from the room.
  - 1. I referred to the printed output (more than one foot back).

never seldom often

2. If I had been allowed to take the paper along, I would have studied it later.

no maybe probably surely

3. I borrowed some one else's paper to study.

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yes no

- 4. Being able to look back at the paper was useful.
  - not at all slightly moderately considerably very much
- The was allowed to refer to the printed output while at the terminal and to take it along after the session.
  - 1, 1 referred to the printed output (more than one foot back).

    never seldom often
  - 2. Being able to look back was useful.
    - not at all slightly moderately considerably very much
  - 3. After the terminal session, how much time did you spend studying the output?
  - 4. Being allowed to take the paper along was useful.
    - not at all slightly moderately considerably very much
  - 5. I found the following more useful for later study.
    - Conte session output about equal



APPENDIX F
EXAMINATION SCORES

		Pretest 1	Pretest 2	Posttest
Group	1	47  30 50 42 35 45	40  25 40 45 37 25	100 98 94 92 87 83 71
Group	2	49 50 15 25 39 34 25 30	25 18 32 32 20 20	67 89 32 80 76 72 64
Group	3	49 46 23 42 46 41	28 43 25 32 22 40	96 93 84 83 82 80 77 68